Mass balance for a water tank. (Part 1: A simple tank)

A basic understanding of Systems thinking is essential to understanding our complex society and everyday lives, and is important to anyone trying to understand systems in the natural sciences, economics, social sciences, etc. In this activity the student is introduced to several fundamental concepts related to all systems. These include equilibrium, time delay or response time, the relationship between system flows and accumulations, and feedback processes. Here we use a water tank with some water volume V, an inflow S, and some possible outflow via a hole in the bottom. This is an excellent analogy to a variety of real world systems. For example: air, water, or soil pollution, hydrology, home energy use, medication dosage rate and blood concentrations, or the buildup of psychological stress during an ongoing stressful situation.

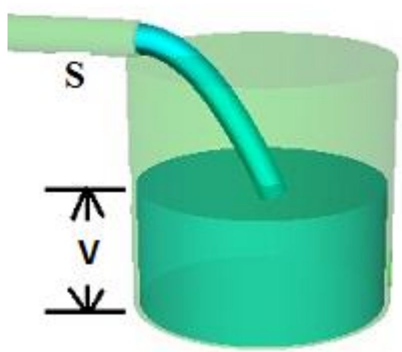


Figure1. A simple water tank with volume V and inflow S.

Part 1: Learning objectives.

1. Students will graph the behavior of a simple water tank system, labeling axes correctly and including the corresponding units.
2. Students will use their graphs and a simulator to compare water levels for different assumed initial volume and inflow rates.
3. Students will develop an equation to describe the time behavior of a simple tank system.

Watch the video [FillingUp]

Imagine a 10 gallon tank that is filled by a faucet. We refer to the faucet as the source of water flowing into the tank, S. If S=2.0 gallon per minute (2.0 gal/min) then the tank will become completely filled in 5 minutes. The amount of water in the tank at any given time will be referred to as the tank volume, V.

If we start with an empty tank and turn on the faucet at a constant flow (S in gallons per minute), then the water volume V at any time t is easy to calculate from

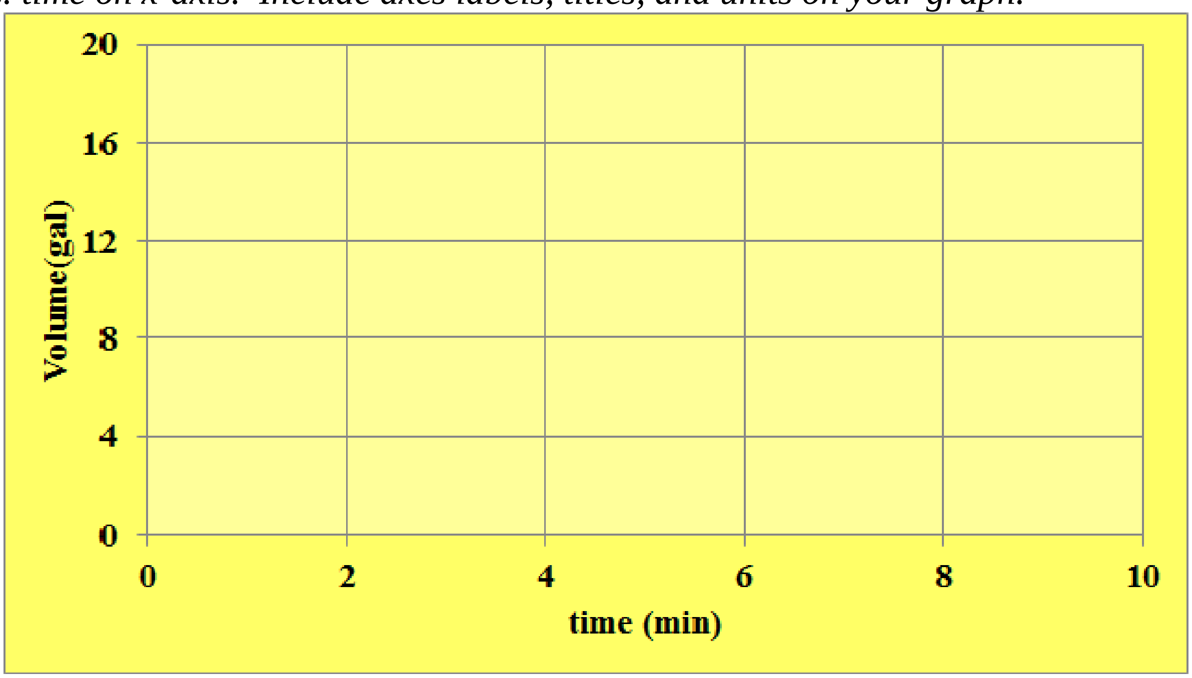
$$V = S\,t \qquad \textbf{Eqn 1}$$

Complete Table 1 below for S=2.0 gal/min (gallons per minute)

Table 1. A simple tank with an inflow of 2.0 gal/min (read as gallons per minute).

Time (min)	**0.0**	**1.0**	**2.0**	**3.0**	**4.0**	**5.0**	**6.0**	**7.0**	**8.0**	**9.0**	**10.0**
Volume (gallons)	**0.0**	**2.0**	**4.0**								

Fill in the table above and then use the set of axes below to make a graph of tank content on y-axis vs. time on x-axis. Include axes labels, titles, and units on your graph.



Note: If you are doing this assignment online you can use a graphing program or do the graph by hand and then take a photo of it and paste it in the space above.
Or easier yet [watch this video for an easy way to graph](#) using the above axes right in this document.

Check your understanding. If the tank starts out empty (V_0=0) and is filled at a rate of S=3.0 gal/min, how much water will be in the tank in 3.0 minutes? The symbol V_0 stands for the initial tank volume at time t=0.

Tank content in 3 minutes will be gallons ______________________

Check point 1 Check Answer Ctrl-click

Question: If your grandma gives you $1000 a month for 10 months, and you don't spend any. How much money do you have in your piggy bank after 10 months?______________________

Question: Say that you start out with $5000 in your piggy bank, and then your grandma gives you $1000 a month for 10 months and you don't spend any. How much money do you have in your piggy bank after 10 months? ______________________

Check point 2 Check Answer Ctrl-click

Does the equation below make sense? ______________________

Balance after 10 months = Starting Balance + Monthly Deposit*10

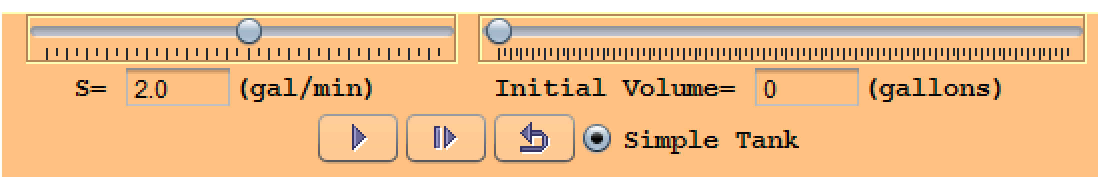
Or in symbolic form B = Bo + MonthlyDeposity*t (t is the time in months, for this case t=10)

Hint: the answer is YES

Using the online Model for a simple tank

Ctrl-Click here to access the WaterTankJAVASimulator

For this part of our activity we will Select the Simple Tank radio button to get only two slider inputs



Objective: Use the Water Tank Simulator to explore how a non-leaky tank fills up over time, and to determine an equation that describes this process.

Start with an empty tank (Vo=0). This is the simple tank with no leak so it should fill up linearly.

Q1: Using Vo=0, and S=4.0 gal/min. How much water is in the tank in 20 min? ***Warning: If you enter numbers into a simulator text box directly, make sure to press the enter key. If the textbox is highlighted, then the numerical value has not been entered.***

Q2: repeat Q1 for S=2.0 gal/min, S=1.0 gal/min, and S=0.5 gal/min (all with V_0=0). You can click the mouse down and move around the graph to read values right off the graph.

S(gal/min)	Vat 20 min (gal)
0.5	
1.0	
2.0	
4.0	

Q3: For the four flow rates {S=0.5gal/min, 1.0 gal/min, 2.0gal/min, and 4.0 gal/min} rank them in order of water content at 20 min from greatest to least.

greatest			least

Q4: If the flow rate is S=4.0 gal/min and Vo=0.0, how much water is in the tank in 10.0 min?

Q5: If the flow rate is S=4.0 gal/min and Vo=20.0 gallons, how much water is in the tank in 10.0 min?

Q6: For the four pairs **(flow rate in gal/min S , initial water volume in gallons V_0)**
A=(3.0, 5), B=(4.0, 0.0), C=(2.0, 30), and D=(0.0, 45)
rank them in order of greatest to least water content at 10.0 min.

greatest			least

Write an equation for the water volume **V** at any time **t** that involves the initial water volume (**Vo**), the flow rate into the tank (**S**), and time (**t**)? Try it and check to make sure it works. Write your equation below. It should only have V, Vo, S and t in it. See grandma/ piggy bank example for hint.

V=____________________________

Questions to test your understanding:

Q7: A simple 100 gallon tank, starts out empty and fills at a rate of 1.5 gallons per minute. How much is in the tank after 20 minutes?

Q8: A simple 100 gallon tank, starts out half full, and fills at a rate of 1.5 gallons per minute. How much is in the tank after 20 minutes?

Q9; A simple 100 gallon tank, starts out three quarters full, and fills at a rate of 1.5 gallons per minute. How much is in the tank after 20 minutes

A graph of tank volume versus time is shown at right.

Q10: At what point is the inflow greatest?
A B C D

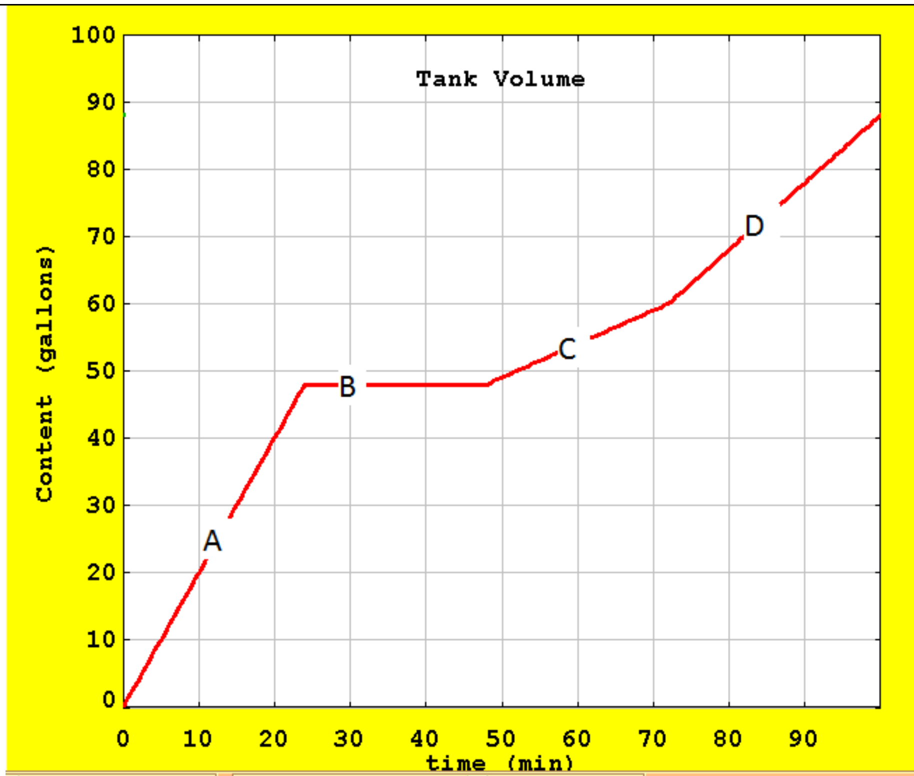
Q11. At what point is the inflow smallest?
A B C D

Q12: Which Rank below corresponds correctly to the correct ranking from least to greatest inflow?

a) A B C D
b) B C D A
c) B D C A
d) C D B A

Q13. Estimate the inflow rate in gallons per minute at point D.

Inflow at D=______________



Part 2. A slightly more complex water tank.

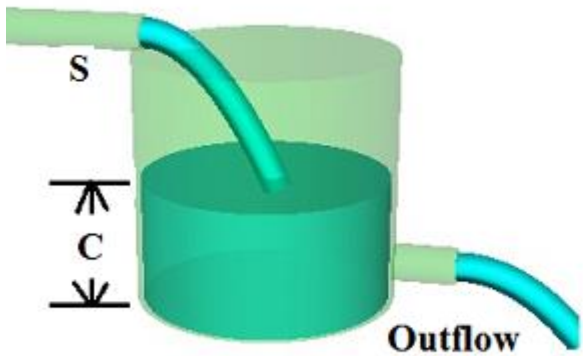


Figure 2. A water tank with content C, inflow S and Outflow. The outflow is directly proportional to the water level or tank content. See more background .

Part 2: Learning objectives.

1. Students will graph the behavior of the more complex water tank system.
2. Students will use their graphs and a simulator to compare water levels for different assumed initial volume, inflow rates, and tank residence times
3. Students will identify equilibrium states.
4. Students will distinguish between short and long residence times.
5. Students will develop an equation to describe equilibrium behavior of the complex tank system.

Watch the video Draining

Definition of shorthand terms:

t- time (in minutes)

V – Water Volume at any time (in gallons)

Vo – initial water Volume at t=0 (in gallons)

Veq- Volume at equilibrium

τ- lifetime (in minutes)

the lifetime is also referred to as response time, delay time constant, or residence time. It reflects how long it will take the system to drop down to 37% of its starting value after all inflow has been turned off. In this case, the lifetime is related to the hole size and viscosity of the fluid in the tank. A small hole in the bottom of the tank corresponds to a longer lifetime than for a large hole.

S- inflow in gallons per minute (gal/min)

Outflow –outflow from the hole

For a leaky tank the outflow from the hole at any time is directly proportional to the Volume V at that time; the fuller the tank, the greater the pressure pushing water out the bottom hole. The constant of proportionality is related to the lifetime (response time) of the tank system via,

$$\text{outflow} = \frac{V}{\tau}$$

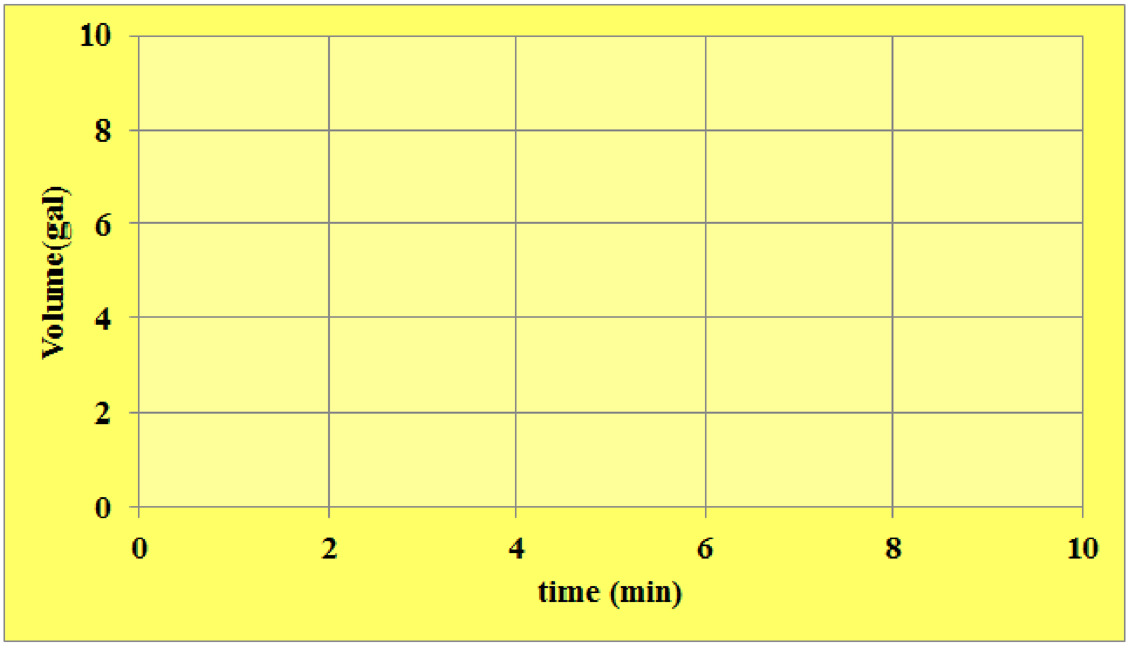
For example, a leaky tank with a lifetime(τ) = 10 min, means that the outflow in gallons per minute is 1/10th of what is in the tank at any time. Or, a simpler way of looking at it is, that after each minute passes, 90% of what was in the tank is still there, the rest flowed out. To fill in the table below you simply multiply the volume at any time by 0.90 to get the next value. If you do this correctly you should end up with 3.5 for the content after 10 minutes.

Table 2. A simple tank with and initial content of 10 gallons, no inflow, and a lifetime of 10 minutes.

Time (min)	**0.0**	**1.0**	**2.0**	**3.0**	**4.0**	**5.0**	**6.0**	**7.0**	**8.0**	**9.0**	**10.0**
Volume (gallons)	**10.0**	**9.0**	**8.1**								

Fill in the table above and then use the axes below to make a graph of tank content on y-axis vs. time on x-axis. Include axes labels, titles, and units on your graph.

Model of mass balance for a water bucket***, heat loss, and banking***

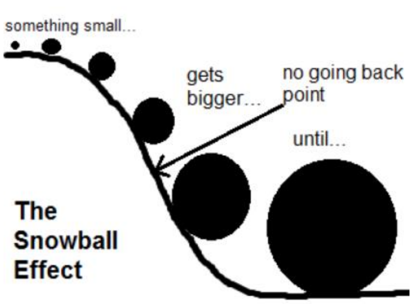


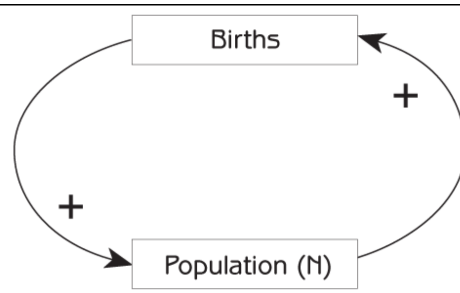
Feedback: a brief introduction.

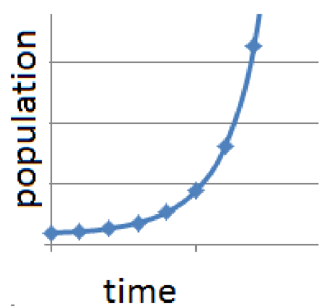
Feedback processes are inherent in most systems. There are two types of feedback processes, positive and negative feedback. The term positive and negative feedback are not related to a judgment regarding the benefits of the feedback, rather positive feedbacks tend to amplify a change initiated within a system and negative feedbacks tend to buffer change. Two examples will clarify.

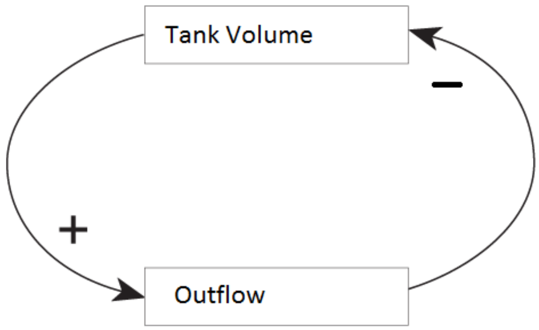
Positive Feedback, species population: Assume a population growth rate OF 2 % per year. When the population is 1 billion for a given year, then the population will grow by 20 million that year, and when the population is 2 billion the growth will be 40 million. This positive feedback is graphically shown in the diagram at right.

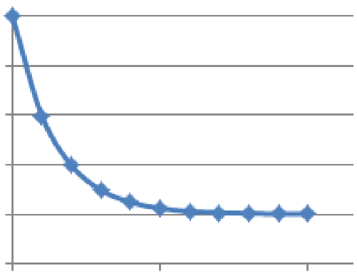
Positive feedback is associated with exponential growth and is sometimes referred to as the snowball effect.











Negative Feedback, the leaky tank: For a leaky tank, with a lifetime of 10 minutes, the outflow rate is 1/10 th of the volume. If the tank has 10 gallons in it, the outflow is 1 gal/min, and if the water volume is 5 gallons, then the outflow is 0.5 gal/min. This negative feedback is graphically shown in the diagram at left.

Negative feedback is associated with stability, and a exponential decay to a new stable state.

Using the online Model for a leaky tank with S=constant

Click here to access the WaterTankJAVASimulator

Objective: Explore how the equilibrium water level depends on the initial water level for a leaky tank.

Watch the videos Filling and draining and TooFull

Q7: Start with Vo=0, lifetime=10 min, and S=4gal/min. What is the final equilibrium water volume?

Q8: Repeat Q7 except use Vo=20gal, 40gal, 60gal, & 100gal. (fill in the table below for your answer)

Vo (gal)	Veq
0	
20	
40	
60	
100	

Check Answer
Ctrl-click

Q9: Does the equilibrium content depend on the initial content?

Objective: Explore how the equilibrium water content depends on the lifetime for a leaky tank. (here we keep the flow rate into the tank fixed at 4 gal/min)

Q10: Using Vo=0 and S=4gal/min and a lifetime of 2 min what is the final equilibrium water content. Repeat using lifetimes of 5, 10, 20, and 25 minutes.

lifetime (min)	Veq
2	
5	
10	
20	
25	

Q11: Does the tank equilibrium content depend on the lifetime?

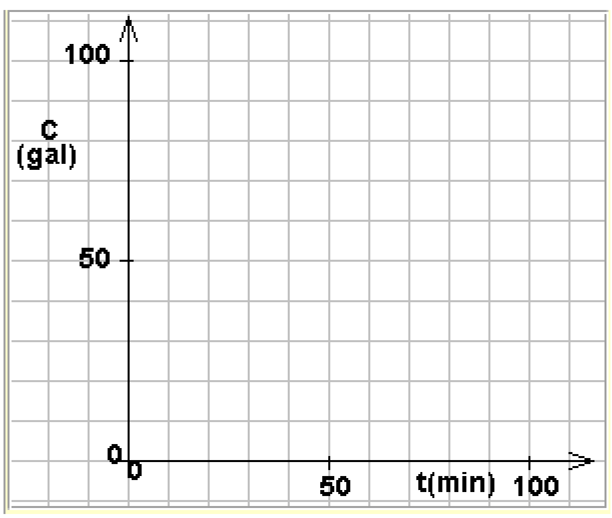
How does doubling the lifetime from 5 to 10 minutes influence the equilibrium content?

How does quadrupling the lifetime from 5 to 20 minutes influence the equilibrium content?

Q12: For tank life-times of 10, 20, 50, & 100 minutes, Rank each tank from highest to lowest equilibrium content assuming all have the same fixed inflow S.

Highest equilibrium			lowest equilibrium

Using Vo=100 gal, S=0.0 and lifetime 10 min sketch the behavior of the water in the tank for a 100 min simulation. Repeat using a lifetime of 25 min. (You can also do a screen capture from the simulation environment with both the 10 min and 25 min lifetime simulations on the same axes and replace the image below).



Q13: For tank lifetimes of 10, 20, 50, & 100 minutes, Rank each tank from most leaky to least leaky.

most leaky			least leaky

Objective: Explore how the equilibrium water content depends on the flow rate into a leaky tank (Source, S) for a leaky tank. (here we keep the lifetime fixed at 10 min)

Q14: Find the equilibrium water content using Vo=0, lifetime=10 minutes, and S=1 gal/min. Repeat this experiment using S = 2, 3, and 4 gallons/minute. { keep Vo=0, lifetime=10 min }

S (gal/min)	Veq
1	
2	
3	
4	

Q15: Does the tank equilibrium content depend on the flow source (S)?

How does doubling S from 1 to 2 gal/min influence the equilibrium content?

How does quadrupling S from 1 to 4 gal/min influence the equilibrium content?

Q16: For S =2 , 4, 6, and 8 gallons/minute, Rank each from highest to lowest equilibrium content.

Highest equilibrium			lowest equilibrium

Q17: Which of these equations best describes the equilibrium content in a tank with an inflow S and a given lifetime?
a. Ceq=S*(lifetime)
b. Ceq=(lifetime)/S
c. Ceq=S/(lifetime)
d. Ceq=S+(Lifetime)

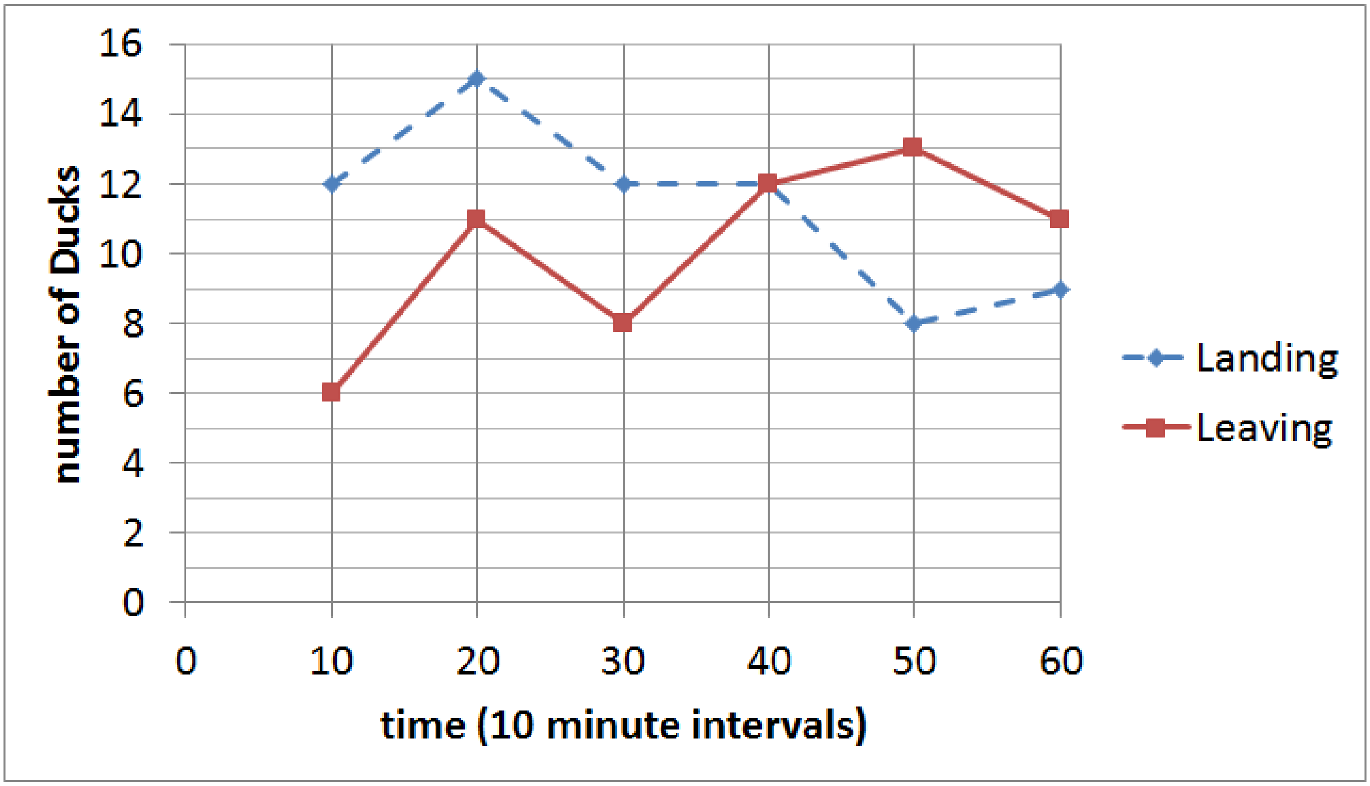
Additional questions to test your understanding.

Q18: At a particular instant the inflow into a tank is 40 gal/min and the outflow is 30 gal/min. At this time the water level in the tank will
a) Increase b) decrease c) stay the same

Q19: At a particular instant the inflow into a tank is 20 gal/min and the outflow is 20 gal/min. At this time the water level in the tank will
a) Increase b) decrease c) stay the same

Q20: At a particular instant the inflow into a tank is 40 gal/min and the outflow is 50 gal/min. At this time the water level in the tank will

a) Increase b) decrease c) stay the same



The figure above shows data for ducks either landing or leaving a pond for six 10 minute intervals over the course of an hour. The number for 10 corresponds to the 0 to 10 minute interval of time, the number at 20 for the 10 to 20 minute time interval, etc.

Q21: What is the greatest number of ducks landing on the pond for any 10 minute interval?

____

Q22: What is the largest increase in total number of ducks on the pond for any 10 minute interval? ___________________________

Q23: At what time (in min) is there the greatest number of ducks on the pond?

a) 0 b) 10 c) 20 d) 30 e) 40 f) 50 g) 60

Q24: At what time (in min) is there the least number of ducks on the pond?
a) 0 b) 10 c) 20 d) 30 e) 40 f) 50 g) 60

C=S t= 3.0 gal/min (3.0 min)= 9.0 gal Ctrl-Click to Return

C=$5000 + $1000 /month (10 months)= $15,000 Ctrl-Click to Return

Q8: Repeat Q7 except use Vo=20gal, 40gal, 60gal, & 100gal. . (fill in the table below for your answer)

Vo (gal)	Veq
0	40
20	40
40	40
60	40
100	40

Ctrl-Click to Return